**LO 1.** Define the standardized (Z) score of a data point as the number of standard deviations it is away from the mean: $Z = \frac{x-\mu}{\sigma}$.

**LO 2.** Use the Z score

- if the distribution is normal: to determine the percentile score of a data point (using technology or normal probability tables)
- regardless of the shape of the distribution: to assess whether or not the particular observation is considered to be unusual (more than 2 standard deviations away from the mean)

**LO 3.** Depending on the shape of the distribution determine whether the median would have a negative, positive, or 0 Z score.

**LO 4.** Assess whether or not a distribution is nearly normal using the 68-95-99.7% rule or graphical methods such as a normal probability plot.

- *Reading: Section 4.1 of Advanced High School Statistics*
- *Video: Normal Distribution - Finding Probabilities - Dr.Çetinkaya-Rundel, YouTube, 6:04*
- *Video: Normal Distribution - Finding Cutoff Points - Dr.Çetinkaya-Rundel, YouTube, 4:25*
- *Additional resources:*
  - *Video: Normal distribution and 68-95-99.7% rule, YouTube, 3:18*
  - *Video: Z scores - Part 1, YouTube, 3:03*
  - *Video: Z scores - Part 2, YouTube, 4:01*
- *Test yourself: True/False: In a right skewed distribution the Z score of the median is positive.*

**LO 5.** Calculate the sampling variability of the mean, the standard deviation of the sample mean, as $SD_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, where $\sigma$ is the population standard deviation.

**LO 6.** Recognize that the Central Limit Theorem (CLT) tells us about the *shape* of the sampling distributions, and that given certain conditions, the distribution of the sample mean will be nearly normal.

- In the case of the mean the CLT tells us that if

  (1a) the sample size is sufficiently large ($n \geq 30$) or
  (1b) the population is known to have a normal distribution

  then the distribution of the sample mean will be nearly normal, centered at the true population mean and with a standard deviation of $\frac{\sigma}{\sqrt{n}}$.

- The larger the sample size ($n$), the less important that the shape of the original distribution becomes, i.e. when $n$ is very large the sampling distribution will be nearly normal regardless of the shape of the population distribution.

**LO 7.** Carry out probability calculations involving a sample mean by first checking that conditions for a normal distribution are met and them performing normal approximation, using the correct standard deviation in the denominator of the Z-score.

- *Reading: Section 4.2 of Advanced High School Statistics*

- *Videos:*
  - *Sample vs. sampling distribution, Dr. Çetinkaya-Rundel (9:30)*
  - *Working with the CLT for means, Dr. Çetinkaya-Rundel (3:16)*
  - *Distribution of the sample mean, YouTube (6:17)*
  - *Central Limit Theorem, Khan Academy (9:49)*
  - *Sampling Distribution of the Mean, Khan Academy (10:52)*
- *Test yourself:*
  1. *Suppose heights of all men in the US have a mean of 69.1 inches and a standard deviation of 2.9 inches. Would a randomly selected man who is 72 inches tall be considered unusually tall? Would it be unusual to have a random sample of 100 men where the sample average is 72 inches?*

**LO 8.** Determine if a random variable follows a binomial distribution. Recall from Chapter 3 that a variable is binomial if the following four conditions are met:

- The trials are independent.
- The number of trials, n, is fixed.
- Each trial outcome can be classified as a success or failure.
- The probability of a success, p, is the same for each trial.

**LO 9.** Calculate the probabilities of the possible values of a binomial random variable using the binomial formula. Recall that the sum of the probabilities must add to 1.

**LO 10.** Calculate the expected number of successes in a given number of binomial trials ($\mu = np$) and its standard deviation ($\sigma = \sqrt{np(1-p)}$).

**LO 11.** When number of trials is sufficiently large ($np \geq 10$ and $n(1-p) \geq 10$), use normal approximation to calculate binomial probabilities, and explain why this approach works.

- *Reading: Section 4.4 of Advanced High School Statistics*
- *Video: Binomial Distribution - Finding Probabilities - Dr.Çetinkaya-Rundel, YouTube, 8:46*
- *Additional resources:*
  - *Video: Binomial distribution, YouTube, 4:25*
  - *Video: Mean and standard deviation of a binomial distribution, YouTube, 1:39*
- *Test yourself:*
  1. *True/False: We can use the binomial distribution to determine the probability that in 10 rolls of a die the first 6 occurs on the 8th roll.*
  2. *True / False: If a family has 3 kids, there are 8 possible combinations of gender order.*
  3. *True/ False: When* $n = 100$ *and* $p = 0.92$ *we can use the normal approximation to the binomial to calculate the probability of 90 or more successes.*

**LO 12.** Recognize that the sample proportion, $\hat{p}$, has a distribution and that its distribution is centered on the true population proportion, $p$.

**LO 13.** Calculate the sampling variability or standard deviation of the sample proportion, as $SD_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

**LO 14.** Recognize that the Central Limit Theorem (CLT) tells us about the *shape* of the sampling distributions, and that given certain conditions, the distribution of the sample proportion will be nearly normal, specifically when $np \geq 10$ and $n(1-p) \geq 10$.

**LO 15.** Carry out probability calculations involving a sample proportion by first checking that conditions for a normal distribution are met and them performing normal approximation, using the correct standard deviation in the denominator of the Z-score.

- *Reading: Section 4.5 of Advanced High School Statistics*
- *Test yourself:*

  1. *Suppose 45% of voters in a particular county favor a proposition for an increase in property tax. If a researcher takes a random sample of 40 people, what is the probability that greater than 50% of the sample will favor the proposition (thus leading the researcher to incorrectly believe that more than half of the voters are in favor)?*